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Linear approximation in a new theory of gravity

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Abstract. A weak-field expansion is developed for a new theory of gravity, based on a Hermitian nonsymmetric $g_{\mu\nu}$. Plane wave solutions exist for both the symmetric and skew parts of $g_{\mu\nu}$. The symmetric part of $g_{\mu\nu}$ is associated with the spin-2 graviton, while the skew part is related to a massless spin-0 boson called the 'skewon'. In the linear approximation the Eötvös experiment is not violated. The one-boson exchange graph is calculated. Only quadrupole and higher poles contribute to gravitational radiation.

1. Introduction

A new theory of gravity has been proposed (Moffatt 1979, 1980), in which the structure of space–time is non-Riemannian and the metric $g_{\mu\nu}$ is Hermitian. The field equations have a rigorous spherically symmetric static solution which is world-line-complete (non-singular) for certain ranges of the two constants of integration. Several other solutions to the field equations, both exact (Moffatt 1979, Kunstatter *et al* 1979, 1980) and approximate (Mann and Moffatt 1981), have been developed.

In this paper, it is shown that radiative solutions to the field equations exist. These solutions do not violate the Eötvös experiment (Braginski and Panov 1971). Plane wave solutions are developed, and an expression for the energy–momentum tensor of such waves is given. The antisymmetric part of $g_{\mu\nu}$ has a physical part of helicity zero; we call the corresponding particle a 'skewon'. For matter sources skewons do not contribute to the energy–momentum tensor.

The field equations of the theory are (Moffatt 1980)

$$g_{\mu\nu,\lambda} - g_{\rho\nu}\Lambda^{\rho}_{\mu\lambda} - g_{\mu\rho}\Lambda^{\rho}_{\lambda\nu} = 0 \tag{1.1}$$

$$\mathbf{g}^{[\mu\nu]}{}_{,\nu} = 4\pi\mathfrak{S}^{\mu} \tag{1.2}$$

$$R_{\mu\nu}(\Gamma) = -\tfrac{2}{3}W_{[\mu,\nu]} + 8\pi(T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}T). \tag{1.3}$$

Here $T_{\mu\nu}$ is the generalised Hermitian energy–momentum tensor, which is the sum of the energy–momentum tensors for matter and for other sources (e.g. electromagnetic), $\mathfrak{S}^{\mu} = \sqrt{-g}S^{\mu}$ is a conserved matter current, corresponding to the number density of fermions (Moffatt 1980), and $\Lambda^{\rho}_{\mu\nu}$ and $\Gamma^{\rho}_{\mu\nu}$ are Hermitian connections, related by

$$\Gamma^{\rho}_{\mu\nu} = \Lambda^{\rho}_{\mu\nu} - D^{\rho}_{\mu\nu}(S) \tag{1.4}$$

where $D^{\rho}_{\mu\nu}(S)$ depends only on S^{μ} and $g_{\mu\nu}$. If $S^{\mu} = 0$, corresponding to the absence of fermion sources, then $\Lambda^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu}$. The Hermitian tensor $R_{\mu\nu}(\Gamma)$ is given by

$$R_{\mu\nu}(\Gamma) = \Gamma^{\beta}_{\mu\nu,\beta} - \tfrac{1}{2}(\Gamma^{\beta}_{(\mu\beta),\nu} + \Gamma^{\beta}_{(\nu\beta),\mu}) + \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} - \Gamma^{\beta}_{\mu\alpha}\Gamma^{\alpha}_{\beta\nu}. \tag{1.5}$$

W_μ is a pure imaginary vector gauge field, formed from an affine connection $W^\lambda_{\mu\nu}$. These are related to $\Gamma^\lambda_{\mu\nu}$ by

$$W^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \tfrac{2}{3}\delta^\lambda_\mu W_\nu \tag{1.6}$$

$$W_\nu = W^\alpha_{[\nu\alpha]}. \tag{1.7}$$

These two equations show that $\Gamma_\mu \equiv \Gamma^\lambda_{[\mu\nu]} = 0$. Finally,

$$\mathfrak{g}_{\mu\nu} = \sqrt{-g}\, g_{\mu\nu} \tag{1.8}$$

where g is the determinant of $g_{\mu\nu}$.

The conservation laws for $\mathfrak{S}^\mu$ and $\mathfrak{T}^{\mu\nu}$ are

$$\mathfrak{S}^\mu{}_{,\mu} = 0 \tag{1.9}$$

and

$$(g_{\nu\rho}\mathfrak{T}^{\nu\alpha} + g_{\rho\nu}\mathfrak{T}^{\alpha\nu})_{,\alpha} - g_{\mu\nu,\rho}\mathfrak{T}^{\mu\nu} + \tfrac{2}{3}W_{[\rho,\nu]}\mathfrak{S}^\nu = 0. \tag{1.10}$$

These equations are respectively related to the identities

$$\mathfrak{g}^{[\mu\nu]}{}_{,\mu,\nu} = 0 \tag{1.11}$$

and

$$(\mathfrak{g}^{\alpha\nu}G_{\rho\nu}(\Gamma) + \mathfrak{g}^{\nu\alpha}G_{\nu\rho}(\Gamma))_{,\alpha} + g^{\mu\nu}{}_{,\rho}\mathfrak{G}_{\mu\nu}(\Gamma) = 0 \tag{1.12}$$

where

$$\mathfrak{G}_{\mu\nu}(\Gamma) = (R_{\mu\nu}(\Gamma) - \tfrac{1}{2}g_{\mu\nu}R(\Gamma))\sqrt{-g}. \tag{1.13}$$

The field equations (1.1)–(1.3) are invariant under the gauge transformation

$$W'_\mu = W_\mu + \lambda_{,\mu} \tag{1.14}$$

where λ is a pure imaginary arbitrary scalar field.

The signature of the metric is $(---+)$ and we use units where $G = c = 1$.

2. Weak-field expansion

We consider a weak gravitational field, with the metric given by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{2.1}$$

where $h_{\mu\nu}$ is a Hermitian tensor, $|h_{\mu\nu}| \ll 1$, and $\eta_{\mu\nu}$ is the usual Minkowski metric diag $(-1, -1, -1, +1)$. We shall solve the field equations to lowest order in $h_{\mu\nu}$. Raising and lowering is done using $\eta_{\mu\nu}$, so that

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} + O(h^2) \tag{2.2}$$

$$h^{\mu\nu} = \eta^{\mu\lambda}\eta^{\sigma\nu}h_{\sigma\lambda} \tag{2.3}$$

and $h^{\mu\nu}$ is a Hermitian tensor.

Equation (1.1) may be solved for $\Lambda^\lambda_{\mu\nu}$:

$$\Lambda^\lambda_{\mu\nu} = \tfrac{1}{2}\eta^{\lambda\sigma}(h_{\sigma\nu,\mu} + h_{\mu\sigma,\nu} - h_{\nu\mu,\sigma}) + O(h^2). \tag{2.4}$$

$D^\lambda_{\mu\nu}$ is found using (1.1), (1.2) and (1.4). A calculation gives

$$D^\lambda_{\mu\nu} = \tfrac{1}{3}(\delta^\lambda_\nu h_{[\mu\beta]}{}^{,\beta} - \delta^\lambda_\mu h_{[\nu\beta]}{}^{,\beta}) \tag{2.5}$$

where we have used the notation

$$h_{[\mu\beta]}{}^{,\beta} \equiv \eta^{\beta\sigma} h_{[\mu\beta],\sigma}. \tag{2.6}$$

From $\Lambda^{\lambda}_{\mu\nu}$ and $D^{\lambda}_{\mu\nu}$ we obtain

$$\Gamma^{\lambda}_{\mu\nu} = \tfrac{1}{2}\eta^{\lambda\sigma}(h_{\sigma\nu,\mu} + h_{\mu\sigma,\nu} - h_{\nu\mu,\sigma}) - \tfrac{1}{3}(\delta^{\lambda}_{\nu} h_{[\mu\beta]}{}^{,\beta} - \delta^{\lambda}_{\mu} h_{[\nu\beta]}{}^{,\beta}). \tag{2.7}$$

Substituting $\Gamma^{\lambda}_{\mu\nu}$ into (1.3), the field equations (1.2) and (1.3) become

$$h_{[\mu\beta]}{}^{,\beta} = 4\pi S_\mu \tag{2.8}$$

$$-\tfrac{1}{2}(\Box h_{\nu\mu} - h_{(\nu\sigma)}{}^{,\sigma}{}_{,\mu} - h_{(\mu\sigma)}{}^{,\sigma}{}_{,\nu} + h_{,\mu\nu} - \tfrac{1}{3} h_{[\mu\sigma]}{}^{,\sigma}{}_{,\nu} + \tfrac{1}{3} h_{[\nu\sigma]}{}^{,\sigma}{}_{,\mu})$$
$$= -\tfrac{2}{3} W_{[\mu,\nu]} + 8\pi(T_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu} T) \tag{2.9}$$

where

$$h \equiv \eta^{\mu\nu} h_{\mu\nu} \qquad T \equiv \eta^{\mu\nu} T_{\mu\nu}. \tag{2.10}$$

To lowest order, the conservation law (1.10) is

$$T^{(\mu\nu)}{}_{,\nu} = 0. \tag{2.11}$$

Equations (1.12) and (1.13) are, to lowest order in h,

$$R_{(\mu\nu)}{}^{,\nu} = \tfrac{1}{2} R_{,\mu}. \tag{2.12}$$

Equations (2.11) and (2.12) are automatically satisfied by (2.9), as is easily verified. Equation (2.8) clearly satisfies (1.9) and (1.11).

A gauge transformation of the form

$$h'_{\mu\nu} = h_{\mu\nu} - \varepsilon_{\mu,\nu} - \varepsilon_{\nu,\mu} \tag{2.13}$$

leaves equations (2.8) and (2.9) invariant. Equation (2.13) is the weak-field form of a general coordinate transformation

$$x'^{\mu} = x'^{\mu}(x^{\mu}) \simeq x^{\mu} + \varepsilon^{\mu}(x) \tag{2.14}$$

and reflects the general covariance of the field equations in the weak-field limit.

By choosing harmonic coordinates

$$g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = 0 \tag{2.15}$$

or

$$h_{(\sigma\mu)}{}^{,\mu} = \tfrac{1}{2} h_{,\sigma} \tag{2.16}$$

the field equations (2.8)–(2.9) become

$$h_{[\mu\nu]}{}^{,\nu} = 4\pi S_\mu \tag{2.17}$$

$$\Box h_{(\mu\nu)} = -16\pi(T_{(\mu\nu)} - \tfrac{1}{2}\eta_{(\mu\nu)} T) \tag{2.18}$$

$$\Box h_{[\mu\nu]} = -\tfrac{4}{3} W_{[\mu,\nu]} - \tfrac{8}{3}\pi S_{[\mu,\nu]} + 16\pi T_{[\mu\nu]}. \tag{2.19}$$

Equation (2.18) is identical to that of general relativity and has solutions of the form (Weinberg 1972)

$$h_{(\mu\nu)}(\boldsymbol{x}, t) = 4\int d^3x' \, \frac{T_{(\mu\nu)}(\boldsymbol{x}', t - |\boldsymbol{x} - \boldsymbol{x}'|) - \tfrac{1}{2}\eta_{\mu\nu} T(\boldsymbol{x}', t - |\boldsymbol{x} - \boldsymbol{x}'|)}{|\boldsymbol{x} - \boldsymbol{x}'|}. \tag{2.20}$$

Equations (2.17) and (2.19) must be solved for $h_{[\mu\nu]}$ and W_μ. The solutions are, with $W_\mu{}^{,\mu}=0$,

$$W_\mu(\boldsymbol{x},t)=-8\pi S_\mu(\boldsymbol{x},t)-6\pi B_\mu(\boldsymbol{x},t) \tag{2.21}$$

and

$$\begin{aligned}h_{[\mu\nu]}(\boldsymbol{x},t)=&-2\int \mathrm{d}^3x'\,\frac{S_{[\mu,\nu]}(\boldsymbol{x}',t-|\boldsymbol{x}-\boldsymbol{x}'|)}{|\boldsymbol{x}-\boldsymbol{x}'|}\\&-2\int \mathrm{d}^3x'\,\frac{B_{[\mu,\nu]}(\boldsymbol{x}',t-|\boldsymbol{x}-\boldsymbol{x}'|)}{|\boldsymbol{x}-\boldsymbol{x}'|}-4\int \mathrm{d}^3x'\,\frac{T_{[\mu\nu]}(\boldsymbol{x}',t-|\boldsymbol{x}-\boldsymbol{x}'|)}{|\boldsymbol{x}-\boldsymbol{x}'|}\end{aligned} \tag{2.22}$$

where

$$B_\mu(\boldsymbol{x},t)\equiv\frac{1}{\pi}\int \mathrm{d}^3x'\,\frac{T_{[\mu\nu]}{}^{,\nu}(\boldsymbol{x}',t-|\boldsymbol{x}-\boldsymbol{x}'|)}{|\boldsymbol{x}-\boldsymbol{x}'|}. \tag{2.23}$$

The interesting feature emerges that $h_{(\mu\nu)}$ and $h_{[\mu\nu]}$ propagate at the velocity of light, while W_μ has a part that propagates and another part that is a contact term.

3. Plane waves

In free space the field equations become

$$\Box h_{(\mu\nu)}=0 \tag{3.1}$$

$$\Box h_{[\mu\nu]}=\tfrac{4}{3}W_{[\nu,\mu]} \tag{3.2}$$

$$h_{[\mu\nu]}{}^{,\nu}=0. \tag{3.3}$$

The general solution of (3.1) is identical to that of general relativity. We get the plane wave solution

$$h_{(\mu\nu)}(x)=e_{(\mu\nu)}\exp(\mathrm{i}k_\lambda x^\lambda)+e^*{}_{(\mu\nu)}\exp(-\mathrm{i}k_\lambda x^\lambda) \tag{3.4}$$

with

$$k_\mu k^\mu=0. \tag{3.5}$$

Because of equation (2.16) we also have

$$k^\mu e_{(\mu\nu)}=\tfrac{1}{2}\eta^{\lambda\beta}e_{(\lambda\beta)}k_\nu=\tfrac{1}{2}ek_\nu. \tag{3.6}$$

Equations (3.2) and (3.3) yield

$$\Box W_\mu=0 \tag{3.7}$$

with $W_\mu{}^{,\mu}=0$, being the gauge chosen for W_μ. Clearly W_μ has Maxwell-type plane wave solutions. However, because of equation (3.2), the Green's function solution for $h_{[\mu\nu]}$ diverges unless $W_\mu=\lambda_{,\mu}$, where λ is a solution of the massless Klein–Gordon equation. Thus (3.2) becomes

$$\Box h_{[\mu\nu]}=0 \tag{3.8}$$

giving

$$h_{[\mu\nu]}=e_{[\mu\nu]}\exp(\mathrm{i}k_\lambda x^\lambda)-e^*{}_{[\mu\nu]}\exp(-\mathrm{i}k_\lambda x^\lambda) \tag{3.9}$$

where (3.5) holds, and

$$k^{\mu}e_{[\mu\nu]}=0 \tag{3.10}$$

because of (3.3).

Equations (3.6) and (3.10) respectively represent four constraints on the polarisation tensor $e_{(\mu\nu)}$ and three constraints on the polarisation tensor $e_{[\mu\nu]}$. However, only two components of $e_{(\mu\nu)}$ and one component of $e_{[\mu\nu]}$ are physically significant. This is seen by considering two fields a_{μ} and b_{μ}, where

$$\Box b_{\mu}=0 \tag{3.11}$$

$$\Box a_{\mu}-(a_{\nu}{}^{,\nu})_{,\mu}=0. \tag{3.12}$$

By writing

$$h'_{(\mu\nu)}=h_{(\mu\nu)}+b_{(\mu,\nu)} \tag{3.13}$$

and

$$h'_{[\mu\nu]}=h_{[\mu\nu]}+a_{[\mu,\nu]} \tag{3.14}$$

we see that equations (3.1)–(3.3) and (2.16) are true for $h'_{\mu\nu}$. The field a_{μ} is a Maxwell-type field, and so has only two degrees of freedom. Four more components of $h_{(\mu\nu)}$ can be made to vanish by (3.13), and two more components of $h_{[\mu\nu]}$ can be made to vanish by (3.14). Thus $h_{(\mu\nu)}$ has only two physically significant components, while $h_{[\mu\nu]}$ has only one component.

As in general relativity, the field $h_{(\mu\nu)}$ has helicity 2, as is easily shown by considering a plane wave in the $\hat{z}$ direction (Weinberg 1972).

Similarly, a rotation about the z axis for a wave of $h_{[\mu\nu]}$ in the z direction gives

$$d'_{\pm}=\exp(\pm \mathrm{i}\theta)d_{\pm} \tag{3.15}$$

$$e'_{[12]}=e_{[12]} \tag{3.16}$$

where

$$e_{[43]}=0 \tag{3.17}$$

$$d_{\pm}=e_{[14]}\mp \mathrm{i}e_{[24]}=-(e_{[13]}\mp \mathrm{i}e_{[23]}) \tag{3.18}$$

and equation (3.10) has been used. Thus $h_{[\mu\nu]}$ has components $d_{\pm}$ of helicity ± 1, and $e_{[12]}$ of helicity zero. However, using (3.12) and (3.14) $e_{[14]}$ and $e_{[24]}$ can be made zero. Thus only $e_{[12]}$ is physically significant, and $h_{[\mu\nu]}$ is a spin-0 field.

It seems natural to call the spin-2 particle associated with $h_{(\mu\nu)}$ the graviton, since $h_{(\mu\nu)}$ is identical to general relativity. We call the spin-0 particle associated with $h_{[\mu\nu]}$ the 'skewon'.

It is necessary to calculate the energy–momentum tensor of the plane waves. This is done using the energy–momentum pseudotensor $t_{\mu\nu}$ in the new theory. Analogous to general relativity (Weinberg 1972, Kunstatter and Moffatt 1979), we have

$$t_{\mu\nu}\simeq\frac{1}{8\pi}(\overset{2}{R}_{\mu\nu}(\Gamma)-\tfrac{1}{2}\eta_{\mu\nu}\overset{2}{R}(\Gamma)) \tag{3.19}$$

where $\overset{2}{R}_{\mu\nu}(\Gamma)$ is $R_{\mu\nu}(\Gamma)$ to $\mathrm{O}(h^2)$. The expression for $t_{\mu\nu}$ is very complicated. However, since $\langle t_{\mu\nu}\rangle$ is all that is measured in practice, we shall calculate it instead. Averaging over space and time in a region much larger than $|\boldsymbol{k}|^{-1}$, we find after a tedious calculation

$$\langle t_{\mu\nu}\rangle=\frac{k_\mu k_\nu}{16\pi}(e^{(\beta\gamma)}e^*{}_{(\beta\gamma)}-\tfrac{1}{2}|\eta^{\beta\gamma}e_{(\beta\gamma)}|^2-e^{[\beta\gamma]}e^*{}_{[\beta\gamma]}). \tag{3.20}$$

This expression is most easily seen to be positive definite using a plane wave travelling in the $\hat{z}$ direction. Using equations (3.15)–(3.18) we get

$$\langle t_{\mu\nu}\rangle=\frac{k_\mu k_\nu}{8\pi}(|e_{11}|^2+|e_{(12)}|^2+|e_{[12]}|^2) \tag{3.21}$$

for the energy–momentum 'Poynting tensor' in the new theory for a wave moving in the $\hat{z}$ direction.

It is seen that plane gravitational waves have, in general, a higher energy content per unit volume than their general relativity counterparts, the extra amount coming from the skewon contribution. It will be shown in the next section that, for spinless matter sources only, the skewon contribution vanishes.

4. Radiation of gravitational waves

In analysing the way in which gravitational waves are generated, it is convenient to consider the sources $T_{\mu\nu}(x)$ and $S^\mu(x)$ in terms of their Fourier components $T_{\mu\nu}(\boldsymbol{x},\omega)$ and $S^\mu(\boldsymbol{x},\omega)$. These are related by

$$T_{\mu\nu}(\boldsymbol{x},t)=\int_0^\infty \mathrm{d}\omega\, T_{\mu\nu}(\boldsymbol{x},\omega)\exp(\mathrm{i}\omega t)+\mathrm{CC} \tag{4.1}$$

$$S^\mu(\boldsymbol{x},t)=\int_0^\infty \mathrm{d}\omega\, S^\mu(\boldsymbol{x},\omega)\exp(\mathrm{i}\omega t)-\mathrm{CC} \tag{4.2}$$

where 'CC' means complex conjugate.

From equation (2.20) it is easy to see that the symmetric part of $h_{\mu\nu}$ is

$$h_{(\mu\nu)}=4\int \mathrm{d}^3x'\,\frac{(T_{(\mu\nu)}(\boldsymbol{x}',\omega)-\frac{1}{2}\eta_{\mu\nu}T(\boldsymbol{x}',\omega))\exp[\mathrm{i}\omega(t-|\boldsymbol{x}-\boldsymbol{x}'|)]}{|\boldsymbol{x}-\boldsymbol{x}'|}+\mathrm{CC} \tag{4.3}$$

where we are considering only a single Fourier component of $T_{(\mu\nu)}$. In the 'wave zone', where the factor ($\hat{x}=\boldsymbol{x}/r$),

$$|\boldsymbol{x}-\boldsymbol{x}'|=r-\hat{x}\cdot\boldsymbol{x}'+\ldots \tag{4.4}$$

and we neglect terms $\mathrm{O}(1/r^2)$. A calculation gives

$$h_{(\mu\nu)}=\frac{4}{r}\exp[\mathrm{i}(\omega t-\omega r)]\int \mathrm{d}^3x'(T_{(\mu\nu)}(x',\omega)-\tfrac{1}{2}\eta_{\mu\nu}T(\boldsymbol{x}',\omega))\exp(\mathrm{i}\omega\hat{x}\cdot\boldsymbol{x}')+\mathrm{CC}. \tag{4.5}$$

This can be rewritten as

$$h_{(\mu\nu)}=e_{(\mu\nu)}(\boldsymbol{x},\omega)\exp(\mathrm{i}k_\lambda x^\lambda)+\mathrm{CC} \tag{4.6}$$

with

$$e_{(\mu\nu)}(\boldsymbol{x},\omega)=\frac{4}{r}(T_{(\mu\nu)}(\boldsymbol{k},\omega)-\tfrac{1}{2}\eta_{(\mu\nu)}T(\boldsymbol{k},\omega)) \tag{4.7}$$

and

$$T_{\mu\nu}(\boldsymbol{k}, \omega) \equiv \int \mathrm{d}^3x' \, T_{\mu\nu}(\boldsymbol{x}', \omega) \exp(\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{x}) \tag{4.8}$$

where k_λ is a wavevector given by

$$k_0 \equiv \omega \qquad \boldsymbol{k} \equiv \omega \hat{x}. \tag{4.9}$$

The large-r limit gives the plane wave solution for $h_{(\mu\nu)}$. The above results are the same as in general relativity (Weinberg 1972).

We now consider W_μ. From § 2, equation (2.21) we see that there is a part of W_μ, namely B_μ, that appears to propagate. Using equations (2.23) and (4.1) we get

$$B_\mu(\boldsymbol{x}, t) = \frac{1}{\pi} \partial^\nu \left(\int \mathrm{d}^3x' \, \frac{T_{[\mu\nu]}(\boldsymbol{x}', \omega)}{|\boldsymbol{x} - \boldsymbol{x}'|} \exp(\mathrm{i}\omega t) - \mathrm{cc} \right). \tag{4.10}$$

In the wave zone we have

$$B_\mu(\boldsymbol{x}, t) = \partial^\nu [b_{[\mu\nu]}(\boldsymbol{x}, \omega) \exp(\mathrm{i}k_\lambda x^\lambda) - \mathrm{cc}] \tag{4.11}$$

with

$$b_{[\mu\nu]}(\boldsymbol{x}, \omega) = \frac{1}{\pi r} T_{[\mu\nu]}(\boldsymbol{k}, \omega), \tag{4.12}$$

and $T_{[\mu\nu]}(\boldsymbol{k}, \omega)$ is given by equation (4.8). Since we neglect all terms of $\mathrm{O}(1/r^2)$ and higher, equation (4.11) becomes

$$B_\mu(\boldsymbol{x}, t) = \mathrm{i}k^\nu b_{[\mu\nu]} \exp(\mathrm{i}k_\lambda x^\lambda) - \mathrm{cc}. \tag{4.13}$$

However, this means that there exist plane waves for W_μ, in contradiction to what was said in § 3. In order to avoid this possibility, we must have either

$$k^\lambda b_{[\mu\lambda]} = 0 \tag{4.14}$$

or

$$T_{[\mu\nu]} = A_{[\mu,\nu]}. \tag{4.15}$$

Equation (4.14) merely says that $T_{[\mu\nu]}{}^{,\nu} = 0$. Thus in the linear approximation we have

$$T_{[\mu\nu]} = A_{[\mu,\nu]} + C_{[\mu\nu]} \tag{4.16}$$

with

$$C_{[\mu\nu]}{}^{,\nu} = 0 \qquad A_\mu{}^{,\mu} = 0. \tag{4.17}$$

A_μ and $C_{[\mu\nu]}$ are sources independent of $h_{\mu\nu}$ or W_μ. We now have

$$W_\mu(\boldsymbol{x}, t) = -8\pi S_\mu(\boldsymbol{x}, t) + 12\pi A_\mu(\boldsymbol{x}, t) \tag{4.18}$$

and so W_μ is a contact term. Thus the vector W_μ *does not propagate* in the linear approximation. In higher orders this is not the case (Moffatt 1980).

Because of (4.16), equation (2.22) now becomes

$$h_{[\mu\nu]} = -2 \int \mathrm{d}^3x' \, \frac{S_{[\mu,\nu]}(\boldsymbol{x}', t - |\boldsymbol{x} - \boldsymbol{x}'|)}{|\boldsymbol{x} - \boldsymbol{x}'|} - 4 \int \mathrm{d}^3x' \, \frac{C_{[\mu\nu]}(\boldsymbol{x}', t - |\boldsymbol{x} - \boldsymbol{x}'|)}{|\boldsymbol{x} - \boldsymbol{x}'|}. \tag{4.19}$$

Suppose $C_{[\mu\nu]}=0$. Then, using equation (4.2) and considering only one Fourier component, we get

$$h_{[\mu\nu]}=-\partial_\nu\left(\int \mathrm{d}^3x'\,\frac{S_\mu(\boldsymbol{x}',\omega)\exp[\mathrm{i}\omega(t-|\boldsymbol{x}-\boldsymbol{x}'|)]}{|\boldsymbol{x}-\boldsymbol{x}'|}\right)$$

$$+\partial_\mu\left(\int \mathrm{d}^3x'\,\frac{S_\nu(\boldsymbol{x}',\omega)\exp[\mathrm{i}\omega(t-|\boldsymbol{x}-\boldsymbol{x}'|)]}{|\boldsymbol{x}-\boldsymbol{x}'|}\right). \tag{4.20}$$

In the wave zone this becomes

$$h_{[\mu\nu]}=e_{[\mu\nu]}(\boldsymbol{x},\omega)\exp(\mathrm{i}k_\lambda x^\lambda)-\mathrm{cc} \tag{4.21}$$

where

$$e_{[\mu\nu]}(\boldsymbol{x},\omega)=\frac{\mathrm{i}}{r}(k_\mu S_\nu(\boldsymbol{k},\omega)-k_\nu S_\mu(\boldsymbol{k},\omega)) \tag{4.22}$$

with

$$S_\mu(\boldsymbol{k},\omega)=\int \mathrm{d}^3x'\,S_\mu(\boldsymbol{x}',\omega)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}'). \tag{4.23}$$

The conservation laws (1.9) and (1.10) yield relations for $T_{(\mu\nu)}$ and S_μ

$$k^\mu T_{(\mu\nu)}=0 \qquad k^\mu S_\mu=0 \tag{4.24}$$

which verify equations (3.6) and (3.10).

The interesting thing about the solution (4.21) is that, although $h_{[\mu\nu]}$ propagates from generation by S_μ, it does not contribute to the energy $\langle t^{\mu\nu}\rangle$ (equation (3.20)). Substitution of equations (4.3) and (4.21) into equation (3.20) yields

$$\langle t_{\mu\nu}\rangle=\frac{k_\mu k_\nu}{16\pi}(e^{(\beta\gamma)}e^*_{(\beta\gamma)}-\tfrac{1}{2}|e|^2). \tag{4.25}$$

We have the important result that skewon fields radiated by S_μ make no contribution to the energy in the plane wave limit of the linear approximation. The post-Newtonian expansion (Mann and Moffatt 1981) of the theory shows that $T_{[\mu\nu]}\propto S_{[\mu,\nu]}$ for matter fields in lowest order. Therefore, as long as $T_{[\mu\nu]}$ is a curl of a matter field only, there can be no skewon contribution to $\langle t_{\mu\nu}\rangle$.

If $C_{[\mu\nu]}\neq 0$, as may well be the case for spinning matter (Yasskin and Stoeger 1979), then there is a skewon contribution to $\langle t_{\mu\nu}\rangle$ generated by the second term of (4.19). The power per unit solid angle is given by (Weinberg 1972)

$$\mathrm{d}P/\mathrm{d}\Omega=r^2x^i\langle t^{i4}\rangle \tag{4.26}$$

and so, using equations (3.20), (4.3), and (4.19) in the wave zone, we get

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega}=\frac{\omega^2}{\pi}(T^{(\mu\nu)}(\boldsymbol{k},\omega)T_{(\mu\nu)}(\boldsymbol{k},\omega)-\tfrac{1}{2}|T(\boldsymbol{k},\omega)|^2-C^{[\mu\nu]}(\boldsymbol{k},\omega)C_{[\mu\nu]}(\boldsymbol{k},\omega)). \tag{4.27}$$

Only $C_{[\mu\nu]}$ contributes to the power and not A_μ.

For $C_{[\mu\nu]}=0$ we get the result of general relativity. Quadrupole and higher poles contribute to the power (Papapetrou 1974), and there is no dipole radiation. The skewon fields have a vanishing contribution to the power output.

It must be noted that for higher orders the $h_{[\mu\nu]}$ and W_μ fields will make contributions to the emitted power. It is only in the linear approximation that $h_{(\mu\nu)}$ dominates for $C_{[\mu\nu]}=0$.

5. The propagator

From equation (4.27), it is easy to deduce that the one-boson exchange graph is given by (Scherk 1979)

$$\mathscr{A}=\frac{16\pi}{q^2}(2(T^{(\mu\nu)}t_{(\mu\nu)}-\tfrac{1}{2}Tt-C^{[\mu\nu]}C_{[\mu\nu]})). \tag{5.1}$$

For spinless particles of mass M, m in the static limit we get

$$\mathscr{A}=\frac{16\pi}{q^2}Mm \tag{5.2}$$

which is identical to the result of general relativity. The Eötvös experiment (Braginski and Panov 1971) is not violated, since only spin-2 gravitons contribute in this case†.

It is possible for other forces to enter in via $C_{[\mu\nu]}$; these forces will be realised through skewon exchange. For example, if $C_{[\mu\nu]}$ is related to the intrinsic spin

$$C^{[\mu\nu]}=\alpha\varepsilon^{\mu\nu\rho\sigma}J_{\rho,\sigma} \tag{5.3}$$

where J_μ is the intrinsic spin pseudovector and α is a constant, then (5.2) becomes

$$\mathscr{A}=\frac{16\pi}{q^2}(Mm+2\alpha^2q^2\boldsymbol{J}\cdot\boldsymbol{j}) \tag{5.4}$$

for two spins $\boldsymbol{J}, \boldsymbol{j}$. We get a spin-dependent force entering in at the one-boson exchange level. The gravitational force is carried by the graviton, and the spin force by the skewon.

We need not commit ourselves to any particular $C_{[\mu\nu]}$ and, in fact, for spinless matter we set $C_{[\mu\nu]}=0$. The fundamental matter fields are $T_{(\mu\nu)}$ and S_μ. S_μ does not contribute to the one-boson exchange graph.

Finally, we write down the propagator:

$$D^F_{\mu\nu\alpha\beta}=\frac{16\pi}{q^2}(2\eta_{\mu\beta}\eta_{\nu\alpha}-\eta_{\mu\nu}\eta_{\alpha\beta}). \tag{5.5}$$

Equation (5.1) then follows from (5.5) and

$$\mathscr{A}=T^{\mu\nu}D^F_{\mu\nu\alpha\beta}t^{\alpha\beta}. \tag{5.6}$$

6. Discussion of results

In the linear approximation to the nonsymmetric Hermitian theory of gravity, the field equations for $g_{(\mu\nu)}$ and $g_{[\mu\nu]}$ decouple. The field equations for $g_{(\mu\nu)}$ correspond to those

† There will be contact interactions of the form $S_\mu S^\mu$ that have to be included in the static limit. Moreover, only the Hermitian (complex) version of the nonsymmetric theory is free of ghosts. For details, see Mann *et al* (1981).

of general relativity for a spin-2 graviton, while the equations for $g_{[\mu\nu]}$ are not Maxwell-type equations, but equations describing a spin-0 massless particle that does not propagate in free space for spinless sources with non-vanishing fermion number current density $\mathfrak{S}^{\mu}$. Since there is no static Coulomb-type force between two S^{μ} currents (Mann *et al* 1981), the Eötvös experiment is not violated, allowing the universal coupling constant a to be of order $\sim 10^{-20}$ cm or less. Since the fermion number F defined by (Moffatt 1980)

$$F = -\mathrm{i} \int \mathfrak{S}^4 \, \mathrm{d}^3 x \tag{6.1}$$

is conserved and the measure of macroscopic coupling is $|l| = a\sqrt{F}$, then for large systems like the sun with $F_{\odot} \sim 10^{57}$ the repulsive (anti-gravity) forces generated in higher orders of approximation can produce significant effects, e.g. prevent gravitational collapse to a black hole for $|l| > 2$ m.

We have also found that gravitational radiation in the new theory is generated by quadrupole and higher multipole moments (no dipole radiation) as in general relativity.

References

Braginski V B and Panov V I 1971 *Zh. Eksp. Teor. Fiz.* **61** 873 (Engl. transl. 1971 *Sov. Phys.-JETP* **34** 464)
Kunstatter G and Moffatt J W 1979 *Phys. Rev.* D **19** 1084
Kunstatter G, Moffatt J W and Savaria P 1979 *Phys. Rev.* D **19** 3559
—— 1980 *Can. J. Phys.* **58** 729
Mann R B and Moffatt J W 1981 *Can. J. Phys.* to be published
Mann R B, Moffatt J W and Taylor J G 1981 *Phys. Lett.* B**97** 73
Moffatt J W 1979 *Phys. Rev.* D **19** 3554, 3562
—— 1980 *J. Math. Phys.* **21** 1798
Papapetrou A 1974 *Lectures on General Relativity* (Dordrecht: Reidel) ch XI, p 158
Scherk J 1979 *Phys. Lett.* **88B** 265
Weinberg S 1972 *Gravitation and Cosmology* (New York: Wiley) ch 10, p 251
Yasskin P B and Stoeger W R 1979 *Preprint* Harvard University